# The bottle-flip challenge demystified: where is the Centre of Mass? 

Paulo Simeão Carvalho ${ }^{1,2}$, Marcelo José Rodrigues ${ }^{2,3}$<br>${ }^{1}$ Departamento de Física e Astronomia, UEC, IFIMUP-IN, Faculdade de Ciências da Universidade do Porto (UP), Portugal.

${ }^{2}$ IFIMUP-IN, Rua do Campo Alegre, s/n, 4169-007 Porto, Portugal.
${ }^{3}$ Middle school of Viso, Porto, Portugal.
psimeao@fc.up.pt ; marcelojrodrigues@sapo.pt


#### Abstract

Bodies can describe very weird trajectories when tossed into the air. The amazing motion of a water bottle-flip is qualitatively described as related to the position of the centre of mass with respect to the body's referential. Although this is rather easy to suspect from simple observation, it is not obvious to verify quantitatively by naked eye.

In this work, we use video analysis and a computational model, to show that the position of the centre of mass (CM) changes in the bottle's referential, while it describes a parabolic trajectory in the laboratory frame of reference. We also show that this behaviour of CM explains the weird motion of the bottle. It can be observed graphically and discussed in the classroom with the students, as it only needs some critical thinking and practically no math.


## Keywords

centre of mass, video analysis, stroboscopic image.

## Introduction

A very popular and viral trick is "the water bottle-flip", where a bottle partially filled with water is tossed into the air with rotation [1,2]. The bottle (apparently) describes a non-parabolic trajectory. The trick is more spectacular when the bottle lands vertically.

An article from Taj Bhutta [3], describes qualitatively the motion in terms of the displacement of the water inside the bottle, which changes the position of the centre of mass (CM) in the bottle's referential. No trick, just physics!

Recently, a paper from Carvalho \& Rodrigues [4] presents a model that allows the calculus of the position of CM in a body, even when it changes in the body's referential during the motion. This model can therefore be used to investigate how the centre of mass displaces during the flip and give a quantitative description of the weird motion of the bottle.

## Experimental

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A plastic bottle was partially filled with coloured water and tossed obliquely, with enough rotation to provide the flip. A video of the bottle-flip was recorded with a SONY Cybershot DSC-RX100 V camera at 100 frames per second (https://youtu.be/7qoCloQn4QM). In order to better describe the trajectories of the whole body, we marked several coloured dots along the bottle's body (figure 1), which were followed during the bottle flip using Tracker software [5].


Figure 1: Bottle partially filled with water and coloured dots in the bottle's body.

## Results and discussion

The video of the bottle-flip was analysed with Tracker software. The stroboscopic image in figure 2 discloses five important steps.


Figure 2: Stroboscopic image of the Bottle during the flip. Five steps are identified.
Step 1 - At the beginning, the water is at the bottom of the bottle so as the CM;

Steps 2 and 3 - When the bottle flips, the water flows through the right side of the bottle's body directly to the top. There is a transfer of water within the bottle, which is responsible for the change in the position of the CM.

Step 4 - Some water also flows from the left side of the bottles' body. Some water can be at the top of the bottle.

Step 5 - The water is nearly equally distributed along the left and right sides of the bottle's body; there is also some water at the top and at the bottom of the bottle.

Plots of the positions of the coloured dots (figure 3) show very weird trajectories, reflecting a complex motion of the bottle. Such behaviour can be understood by considering that the CM moves along the bottle during the flip, because of the water displacement inside the bottle [3]. However, it must be proved.


Figure 3: Plots of the positions of the coloured dots during the whole motion of the bottle.
To do so, we selected some of the coloured dots (namely the white, yellow, green, red, blue ant top red) to run the computational model [4] and calculate the coordinates ( $x_{\mathrm{CM}}, y_{\mathrm{CM}}$ ) of the CM. The model imposes that $x_{C M}(t)$ and $y_{C M}(t)$ are described respectively by linear and parabolic representations, which correspond to the uniform and uniformly accelerated motions, respectively.

Figure 4 presents the computed positions of $x_{C M}$ and $y_{C M}$ as a function of time, as well as depicts those corresponding to the coloured dots of the bottle. At the beginning of the motion (left side on the graphs), the coordinates of CM stand about the central yellow dot. As the flip takes place, the coordinates of CM move upwards in the bottle referential, and at the end of the motion (right side on the graphs), the CM lies between the upper yellow and blue dots. But why does the CM move that way?

When a bucket full of water is rotated horizontally, the water is squeezed to the bottom of the bucket by a local centrifugal force. This kind of force is also felt by the people inside a vehicle when it turns. The higher the speed or the lower the curvature radius, the stronger is the centrifugal effect in the local reference
frame. In the bottle-flip motion, when the bottle leaves the hand (step 1) the water is already being pressed against the bottom of the bottle; but immediately after, when the flip occurs (steps 2 and 3 ), the free axle of rotation is transferred from the bottle's cover (where the hand was) to the centre of mass and consequently the water flows throughout the bottle's body. As CM is somewhere in the middle of the water, the local centrifugal force pushes the water both to the top and to the bottom of the bottle. So, during the fall of the bottle (steps 4 and 5) the CM rises in the bottle's referential because the water is still moving away from the centre of rotation. This means that the bottle must have enough water to locate the CM inside the liquid, but at same time enough "empty" space in the bottle to allow the water to dislocate.


Figure 4: Plots of the coloured dots of the bottle and of the computed coordinates of $\mathrm{CM}\left(x_{C M}, y_{C M}\right)$ as a function of time. The plots have the same colour of the corresponding dots. The first letter of each dot's legend is the position of the dot along the bottle (C-centre; R -right; L-left); the second letter is the colour of the dot (W-white; Y-yellow; G-green; R-red; B-blue. Rup is the upper red dot, TOP is the black dot at the top of the bottle and CM-fit is the computed CM position.

This amazing relative displacement of CM during the flip confirms the qualitative descriptions [1-3] and can be clearly observed in figure 5, where we present simultaneously the stroboscopic motion of the bottle and the positions of CM.

The debate of this motion has a high potential in education. For example, it is important to remember students that in fact, the CM does not dislocates through the bottle, but it is the bottle that moves way from the CM. This effect can be seen in the stroboscopic image of figure 5 at the moment when the bottle stops moving to the left and it begins to fall more vertically. Teachers may use this example to discuss with their students how and why the CM influences the trajectories of bodies or systems of bodies. It can be explored in many contexts where similar motions can be found, such as sports, astronomy or entertainment.


Figure 5: Stroboscopic image of the Bottle during the flip. The centre of mass is represented by pink.

## Conclusions

In this study we show quantitatively that the amazing behaviour observed in the bottle-flip challenge, is due to the displacement of the CM in the referential of the bottle. The position of the centre of mass was computed from a computational model and depicted in a stroboscopic image of the bottle-flip.

This very popular entertaining "game" can be very useful for educational purposes. Indeed, teachers may promote discussions with their students about why the position of the centre of mass can induce weird trajectories in the bottle. Teachers can also recommend their students to study how these trajectories vary with the volume of water inside the bottle. This can be a good strategy to involve the students in a simple but stimulating investigation, and for understanding how athletes perform their jumps (diving, triple jump, etc.) in order to increase their performances.

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