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How to determine the Centre of Mass of bodies from Image Modelling

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Abstract

Image Modelling is a recent technique in physics education that includes digital tools for image treatment and analysis, such as digital stroboscopic photography (DSP) and video analysis software. It is commonly used to analyse the motion of objects. In this work we show how to determine the position of the centre of mass of objects with either isotropic or anisotropic mass density, by video analyses as a Video Based Experimental Activity (VBEA). Strobe imaging is also presented in an educational view, helping students to visualize the complex motion of a rigid body with heterogeneous structure. As an example, we present a hammer tossed with translation and rotation. The technique shown here is valid for almost any kind of objects and it is very useful to work with the concept of centre of mass.

Keywords

Image Modelling, centre of mass, video analysis, stroboscopic image.

Introduction

The study of the motion of large bodies is of great practical interest. It is included in all physics curricular programs, especially in high school where the concept of centre of mass (CM) is an important topic, because it helps to simplify the complex motion of rigid bodies. In schools, teachers usually explain this abstract concept only theoretically, or use statics experiments about the definition of centre of gravity concept. A common experimental technique consists in drawing two vertical gravity lines throughout the body, and the centre of gravity will be at the intersection of these lines.

In this work we propose a new technique to determine the centre of mass of almost any object (either with isotropic or anisotropic mass density) which also acts as an interactive approach for students to learn this concept. We use as an example the motion of a hammer, launched obliquely and with rotation.

The motion of the body is analysed by Image Modelling, namely by digital stroboscopic photography (DSP) [1,2] and by video analysis with *Tracker* software [3]. We observe experimentally the trajectory of several points marked along the body of the hammer and identify the position of the centre of mass from the experimental results.

The motion of bodies and the centre of mass

By tossing obliquely a small, rigid and isotropic mass density sphere in the air with negligible air resistance, we notice that it describes a parabolic trajectory, and because of its symmetry we can ignore its rotation. But if we toss an anisotropic mass density body like a hammer with an heterogeneous structure, at first sight its trajectory cannot be described in a simple way. The hammer's motion is tottering and all points of the object describe distinct trajectories. These points don´t follow a parabolic trajectory because when the hammer rotates, each one moves around a particular point of the body, called the Centre of Mass (CM).

The centre of mass represents the average position (of the discrete mass distribution) that makes up the body. This point has some particular characteristics: it is the single point of the body, after tossed obliquely, whose trajectory is always a parabola, and whose vertical position as a function of time is described by a quadratic function; when the body rotates the CM is always within the rotation axis.

The position of the centre of mass ($\vec{r}_{CM}$) of a body is calculated from equation (1), which takes into account the distribution of the mass in the body.

$$\vec{r}_{CM} = \frac{1}{M}\int \vec{r}\,.\,dm \qquad (1)$$

For bodies with symmetric shape and isotropic mass density, the centre of mass coincides with the geometric centre of the body. However, for bodies with anisotropic mass density and/or with irregular geometric shapes, the analytical determination of the position of CM can be very complex. In the particular case of the hammer, the centre of mass is closer to the more massive end.

The kinematic model of the CM states that in the absence of air resistance and when a body is only exposed to gravity, the equation for the trajectory of the CM is given by the parabolic equation [4]:

$$\Delta y = \Delta x \tan\theta_0 - \frac{g\,(\Delta x)^2}{2.(v_0\cos\theta_0)^2} \qquad (2)$$

where Δy and Δx are respectively the coordinate positions changes in the trajectory along the vertical (y) and the horizontal (x), θ_0 is the tilt angle of launch, v_0 is the initial speed and g is the local gravitational acceleration. According to this model, the vertical component of the CM is described, as a function of time, by the well-known quadratic equation:

$$y = y_0 + v_0\,t\sin\theta_0 - \frac{g}{2}\,t^2 \qquad (3)$$

where y_0 represents the starting position of the CM in the vertical direction, and the horizontal component of the CM is described, as a function of time, by the well-known linear equation:

$$x = x_0 + v_0\,t\cos\theta_0 \qquad (4)$$

where the x_0 represents the starting position of the CM in the horizontal direction.

Centre of Mass *versus* Centre of Gravity

Usually the centre of mass of rigid bodies is identified by making use of another physical concept: the centre of gravity of the body (CG). The centre of gravity is the point through which we can consider the gravitational force to act. This force is the sum of the gravitational forces acting on the discrete elements of the body. If we consider the acceleration of gravity (g) constant, i.e. the same g acting on every single particle within the body, then the centre of gravity is coincident with the respective centre of mass. So, it is very common to determine the position of the CM of extended bodies by means of static techniques used to determine the centre of gravity.

There are many static techniques to determine the CG of a body, e.g. by the interception of two vertical lines corresponding to two static equilibrium body positions. Figure 1 shows a common way for finding the CG of a broom. In this example the CG is within the body, but there are other cases where the CG is outside the body such as in a boomerang or in a very popular equilibrium setup composed of two forks (figure 2).

Figure 1: Estimated longitudinal position of the CG of a broom, obtained from an unstable static equilibrium technique used to find the centre of gravity. The CG is along the vertical line that crosses the wooden stick (the equilibrium is unstable because de CG is above fulcrum).

Figure 2: Estimated vertical position of the CG of the system consisting of two forks. The CG is outside the system in the intersection between the vertical gravity line that crosses the wooden stick and the forks plane, somewhere below the fulcrum.

In this work we will consider that CM and CG are coincident since g is assumed nearly constant.

In the absence of gravity, we can still determine CM by finding the interception of two different free rotating axis in the body. This is a dynamic technique based on the fact that all bodies will freely rotate around an axis that contains the body's CM. This technique can be implemented in space, by applying a torque to the body without conferring a translational movement, but here on Earth we have to deal with the gravity force that compels the objects to fall.

The technique here presented combines the centre of gravity concept with the dynamic rotation technique. As we show in the following, the technique will be useful for the identification of the CM position in a particular rotating plane of the body, confirming that the dynamic study of a seemingly complex motion can be described with great simplicity from the concept of CM.

Description of the technique

For the determination of the CM of a hammer, we tossed it obliquely with rotation and filmed its motion with a Canon EOS 5D Mark III photo camera, at a rate of 60 frames per second. In order to better describe the trajectories of the whole body, we marked several coloured dots along the hammer's body and listed them from A to G (Figure 3).

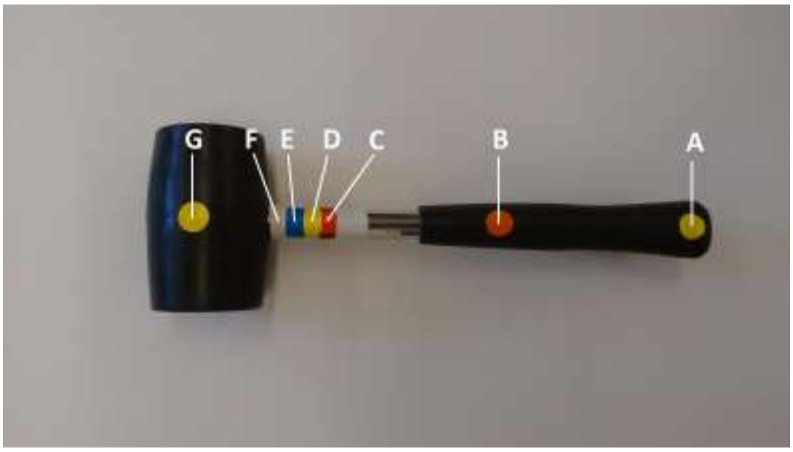

Figure 3: Points distributed over the hammer in order to study their trajectories.

Figure 4 is a strobe image of the motion built with VirtualDub and ImageJ software [5,6], where we can immediately identify that the different parts of the hammer describe distinct trajectories. Despite the different trajectories of each colour mark in the hammer, globally the image shows a parabolic-like symmetry.

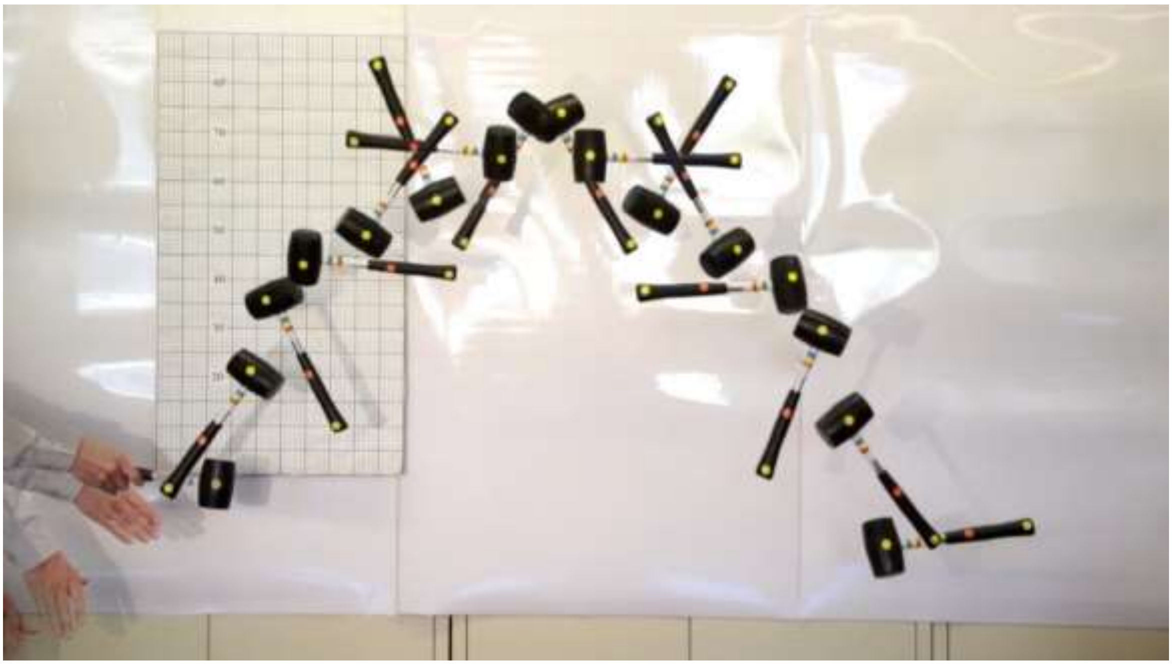

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Figure 4: Strobe image of the oblique movement of a hammer with rotation, made at a capture rate of 20 frames per second. A parabolic-like symmetry can be identified in the image.

Results

The video was analysed with Tracker software and the trajectories of those points were disclosed. The results of the video analysis are presented in Figures 5(a) to 5(g). Undoubtedly, the trajectories of each marked point on the hammer are quite different. The more nearer to the CM are the marked points the more closer to a parabola are the trajectories. The marked point whose trajectory is more closely to a parabola is point E, as we can find in figure 5e.

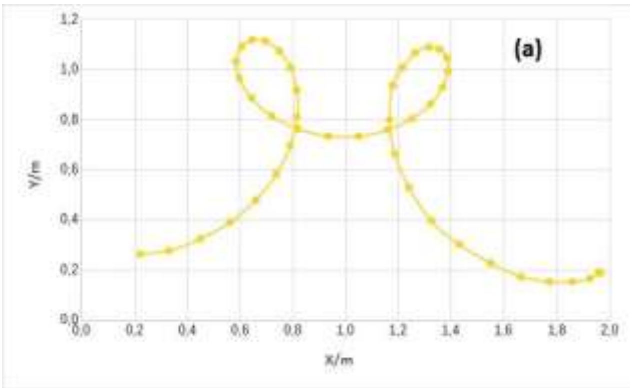

Figure 5(a): Trajectory described by point A, at one end of the hammer.

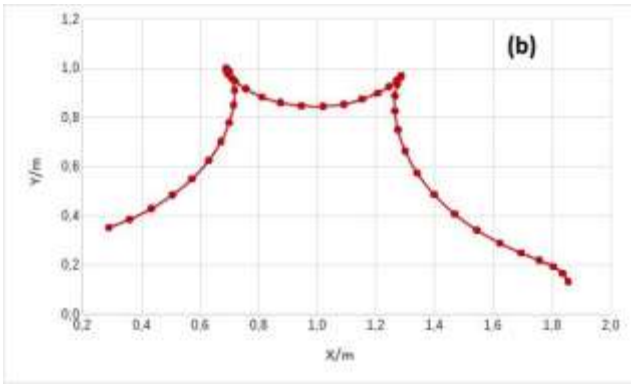

Figure 5(b): Trajectory described by point B.

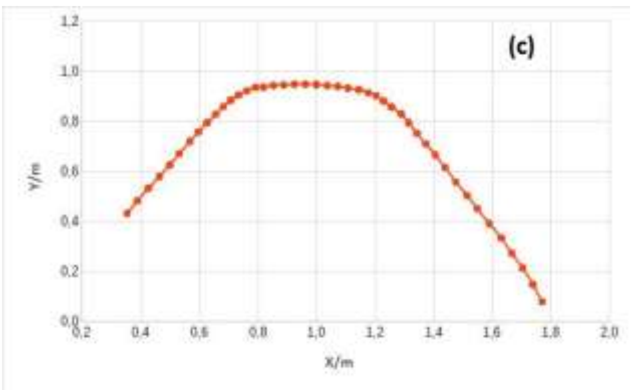

Figure 5(c): Trajectory described by point C.

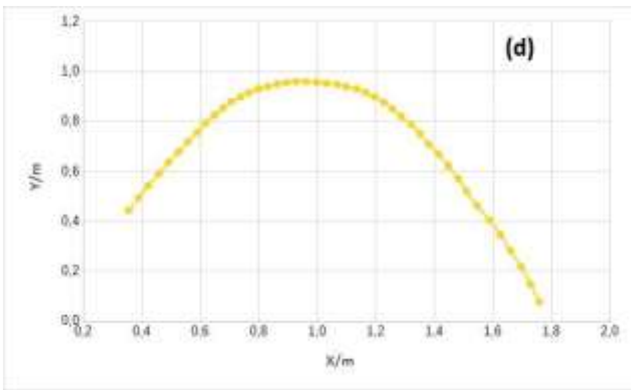

Figure 5(d): Trajectory described by point D.

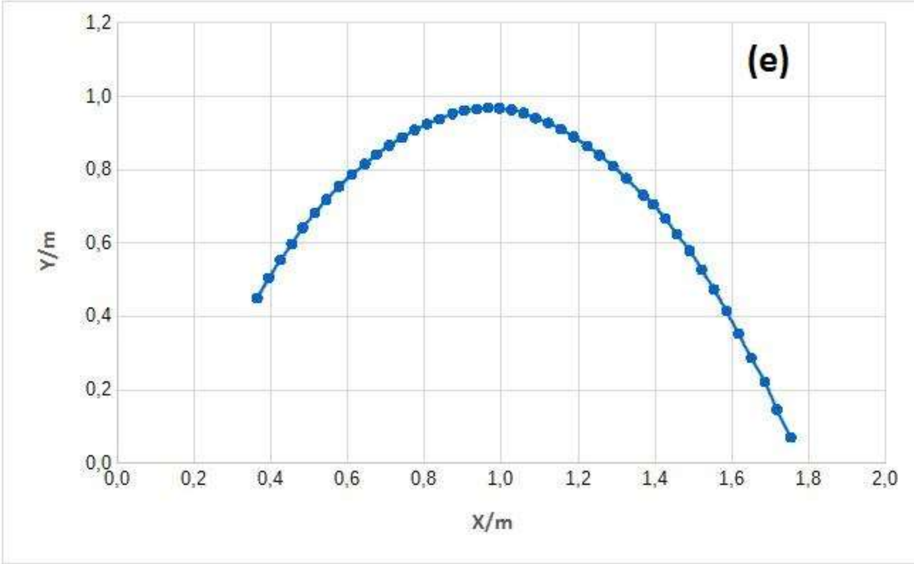

Figure 5(e): Trajectory described by point E. The trajectory shown is close to a parabola.

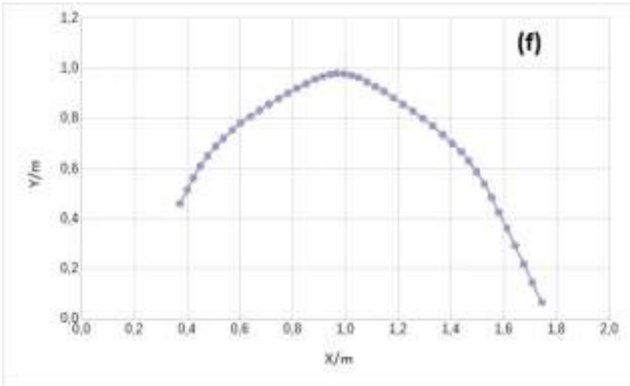

Figure 5(f): Trajectory described by point F.

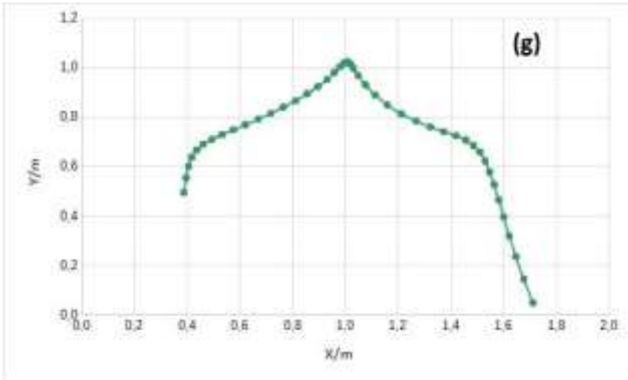

Figure 5(g): Trajectory described by point G, at the other end of the hammer.

Observing the different graphs we can see several points describing parabolic-like trajectories. However, the trajectory of point E is the one that best matches a parabola as described by equation (2), and it is also the only point in the body whose vertical and horizontal coordinates are more approximately described by equations (3) and (4).

Figure 6 presents a fit of a quadratic expression (3) to y for point E as a function of time, found by *DataTool* module of Tracker.

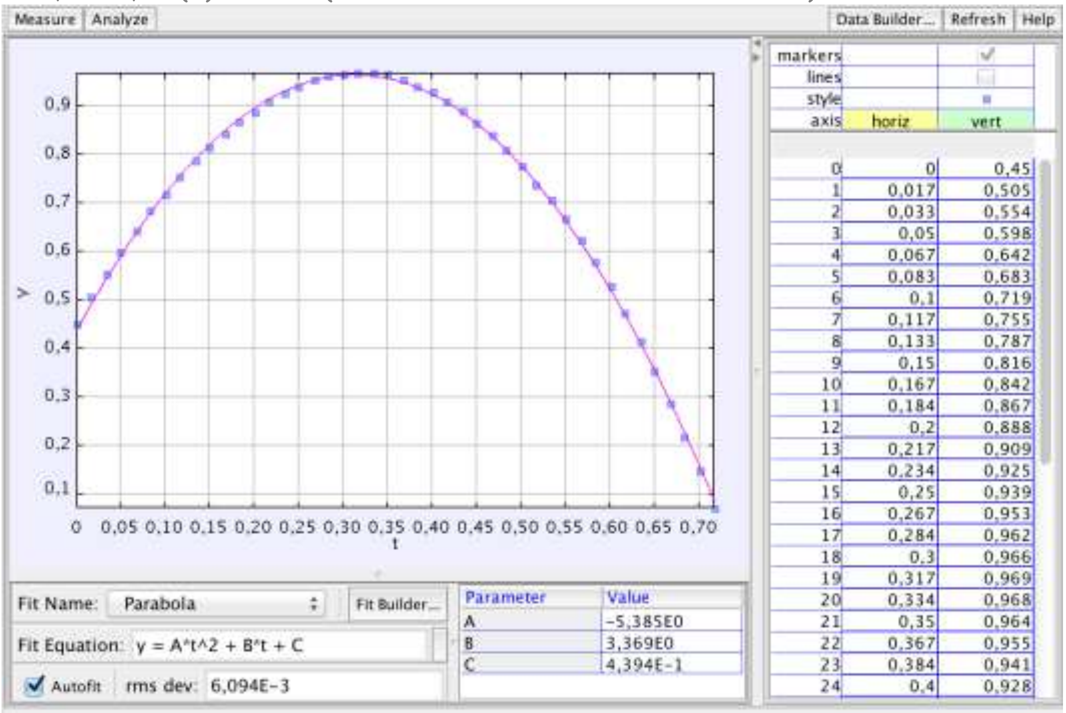

Figure 6: Parabolic fit to *y* positions of point E as a function of time, found by *DataTool* module of Tracker.

We can do an analysis of the quality of the fit by determining the value of the gravitational acceleration *g* from the quadratic term of the fit equation. From the value of parameter A in figure 6 and from equation (3), then the value obtained is g = -10.77 m/s^2. Comparing this with the standard gravitational acceleration on Earth at sea level, g_0 = -9.81 m/s^2, it turns out that the experimental error is approximately 10%; for a complex motion like this and within the approaches assumed by the technique in identifying the most approximated position of CM in the body, this is a fairly acceptable value. One can therefore conclude that the fit is good and strongly suggests point E is located near the CM of the hammer.

In a more general situation, the analysis of all marked points of a body should include both fittings to equation (3) in order to find the best *g* value, and to equation (4) to find the best linear relation, especially when there are a lot of marked points (near the CM) with a good parabolic-like trajectory. The combination of these two information increases the accuracy in the determination of the best point corresponding to the CM.

Conclusions

The educational approach using Image Modelling turns easy the identification of the position of the centre of mass of bodies, because the CM is the only point of a body that always describes a parabolic trajectory when it also has rotational movement. However, this technique is only adequate when the CM is located inside the body and points can be signed and tracked.

With Image Modelling, teachers can create interactive and engaging strategies with their students to determine the position of the CM of bodies. Videos can be recorded not only by teachers, but also by students to increase their engagement and interactivity within the learning process. The technique can be applied to almost any undeformable body, and used to explore the kinematic characteristics of the CM.

Teachers can also develop inquiry activities while teaching mechanics, using everyday contexts and let students interpret counter-intuitive problems. The approach here described allows the study of real and apparently complex motions of bodies, either in the classroom as a practical work, or outside the classroom as educational enrichment tasks.

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References

[1] Dias, M.A., Barros, S.S., Amorim, H.S., "Produção de fotografias estroboscópicas sem lâmpada estroboscópica", *Caderno Brasileiro de Ensino de Física*, 26 (3), 492-513 (2009).

[2] Dias, M.A., "*Utilização de fotografias estroboscópicas digitais para o estudo da queda dos corpos*", MSc. Thesis, Rio de Janeiro: UFRJ (2011) (available at http://www.if.ufrj.br/~pef/producao_academica/dissertacoes/2011_Marco_Adriano_Dias/dissertacao_Marco_Adriano_Dias.pdf , retrieved in 05/15/2015).

[3] Brown, D., Cox, A.J., "Innovative Uses of Video Analysis", *The Physics Teacher*, 47, 145-150 (2009).

[4] Young, H.D., Freedman, R.A. "*University Physics*" (9th edition), Addison-Wesley, New York, 69-70 (1996).

[5] VirtualDub. http://www.virtualdub.org/, retrieved in 07/06/2015.

[6] ImageJ. http://imagej.nih.gov/ij/ , retrieved in 07/06/2015.

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