The problem of a ladder leaning on a wall: experimental investigation of its static equilibrium and dynamics using software Tracker and modelling/simulating using software GeoGebra.

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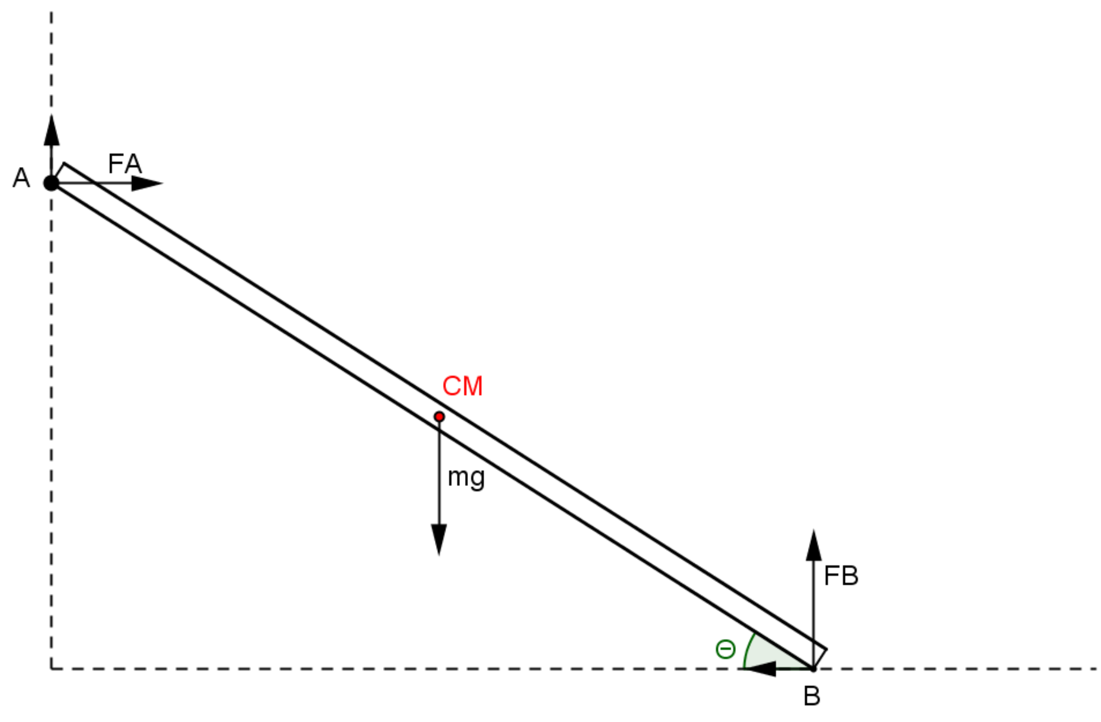
Experiment

- a high-speed photography (220 frames/s)
- a rigid **meter stick** with (or without) a low-friction pulley attached to one end



Modelling the dynamics of the "ladder" leaning against a wall

- rigid body
- friction coefficients of the ladder against the wall μ_w and against the floor μ_f are constant: *a)* $\mu_w = \mu_f = \mu$ *or* *b)* μ_w=0; $\mu_f \neq 0$
- initial conditions are consistent with the top of the ladder being on the verge of slipping downward
- center of gravity of the "ladder" is at the middle.

Static equilibrium: $\begin{cases} \sum F_x = 0 \\ \sum F_y = 0 \\ \sum \tau = 0 \end{cases}$

(on the verge of slipping downward)

- $\theta_{critical} = arctan\left(\frac{1-\mu_w\mu_f}{2\mu_f}\right) = \theta_0; \quad \dot{\theta} = 0$
- $F_A = \frac{\mu_f mg}{\mu_f\mu_w+1}; F_B = \frac{mg}{\mu_f\mu_w+1}$

Dynamic equilibrium: $\begin{cases} \sum F_x = m\ddot{x} \\ \sum F_y = m\ddot{y} \\ \sum \tau = -I\ddot{\theta} \end{cases}$

- $\boldsymbol{t_s}$: instant just before the ladder loses contact with the wall
- $\boldsymbol{t_e}$: the end of the movement
- time intervals separating movement: $\boldsymbol{I_1} = [0, t_s]; \boldsymbol{I_2} = [t_s, t_e]$
- $F_A(\boldsymbol{t_s}) = 0 \qquad \theta(t_e) = 0$

$$\Rightarrow$$

$t \leq t_s$ *(1 degree of freedom)*

$$\frac{d^2\theta}{dt^2} = -\frac{3}{L}\frac{2\mu g \sin\theta - g\cos\theta + \mu^2 g\cos\theta - \mu L\left(\frac{d\theta}{dt}\right)^2}{\mu^2 - 2}$$

$$F_A = -\frac{m}{2(\mu^2 - 2)}\left[3g\sin\theta\cos\theta + L\mu\sin\theta\left(\frac{d\theta}{dt}\right)^2 - 2L\cos\theta\left(\frac{d\theta}{dt}\right)^2 - 2\mu g + 3\mu g(\cos\theta)^2\right]$$

$$F_B = \frac{m}{2(\mu^2 - 2)}\left[-3\mu g\sin\theta\cos\theta + 2L\sin\theta\left(\frac{d\theta}{dt}\right)^2 + L\mu\cos\theta\left(\frac{d\theta}{dt}\right)^2 - 4g + 3g(\cos\theta)^2\right]$$

$t \geq t_s$*(2 degrees of freedom)*

$$\frac{d^2\theta}{dt^2} = \frac{3\left[2\mu g\sin\theta + L\sin\theta\cos\theta\left(\frac{d\theta}{dt}\right)^2 - \mu L\sin\theta^2\left(\frac{d\theta}{dt}\right)^2 - 2g\cos\theta\right]}{L(1 + 3\cos\theta^2 - 3\mu\cos\theta\sin\theta)}$$

$$\frac{d^2x}{dt^2} = \frac{\mu\left(2g - L\sin\theta\frac{d\theta}{dt}\right)}{2(3\mu\cos\theta\sin\theta - 3\cos\theta^2 - 1)}$$

$$F_A = 0$$

$$F_B = \frac{1}{2}\frac{m\left(2g - L\sin\theta\left(\frac{d\theta}{dt}\right)^2\right)}{(1 + 3\cos\theta^2 - 3\mu\cos\theta\sin\theta)} \quad \Rightarrow \quad \text{(numerical calculations)}$$

Experimental data

- $L = 1.005\ m$
- $m = 0.23572\ kg$
- $\theta_0 = 0.69\ rad$
- $\mu = 0.43$

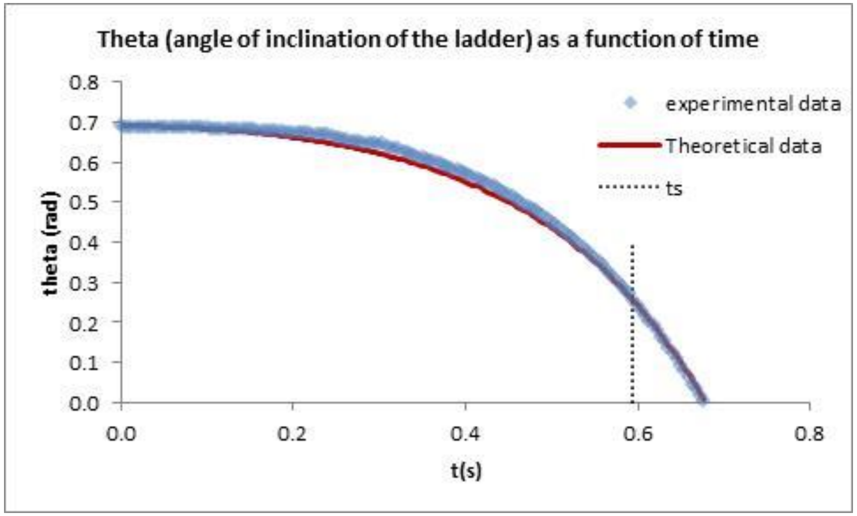
Results obtained using *Tracker*

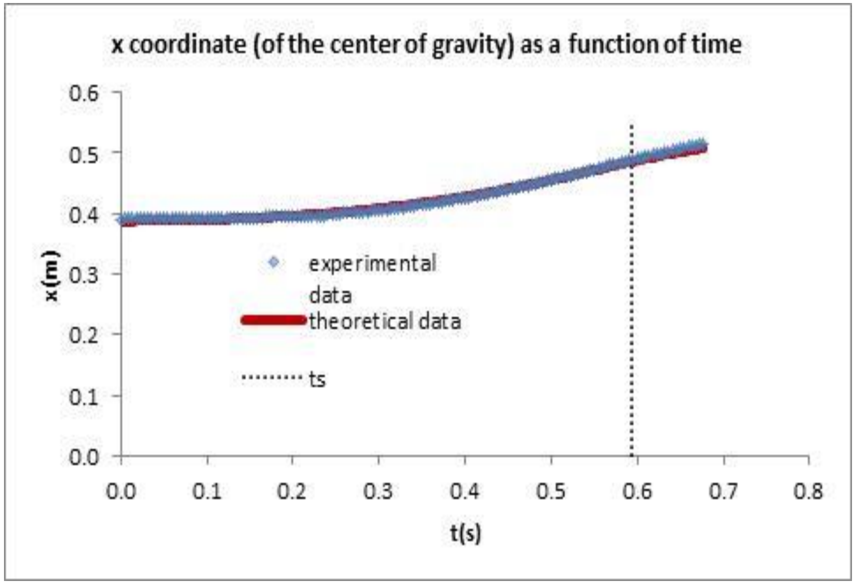
Numerically calculated time critical values

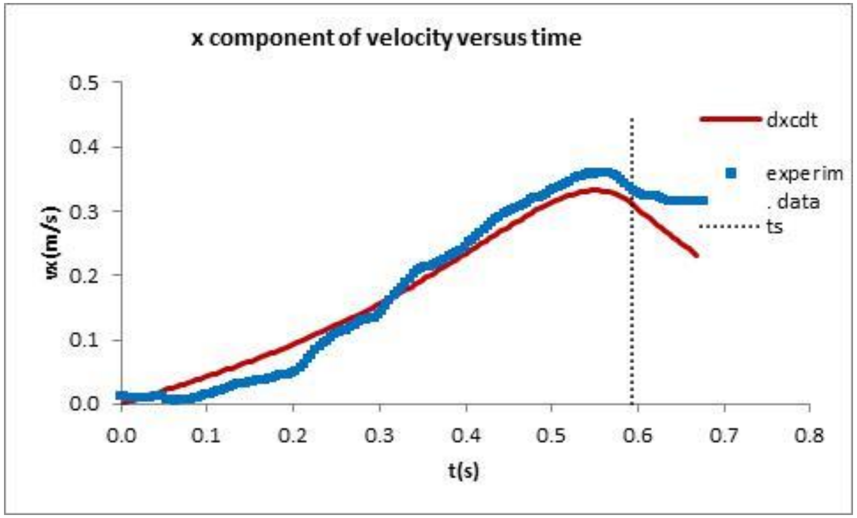
- $\boldsymbol{t_s}$: instant where the ladder loses contact with the wall.
- $\boldsymbol{t_e}$: final instant.
- $\boldsymbol{t_{xmax}}$: instant where the x component of center of gravity velocity reaches a maximum.

$$t_s = 0.592574\ s$$
$$t_e = 0.6791960\ \text{s}$$
$$t_{xmax} = 0.551314\ \text{s}$$
$$\theta(t_s) = 0.259555\ rad$$
$$\frac{d\theta}{dt}(t_s) = -2.41396\ rad/\text{s}$$
$$x(t_s) = 0.4856683\ m$$

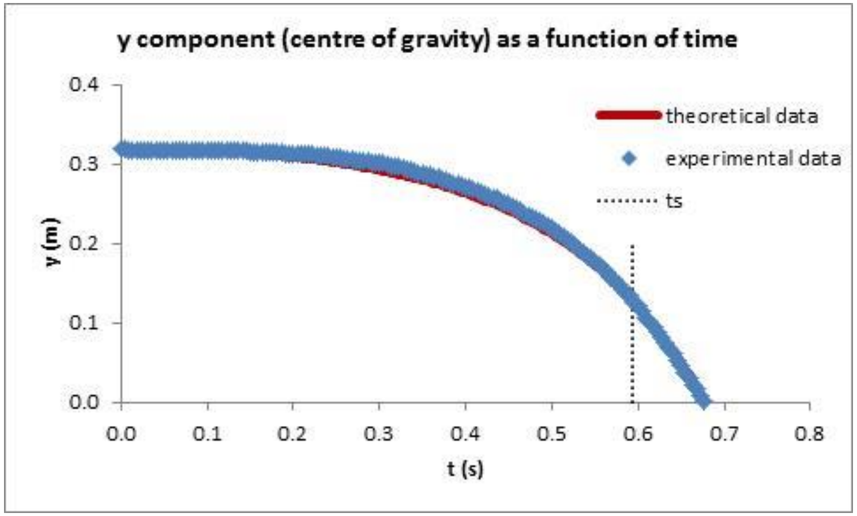
Theoretical data *versus* experimental data

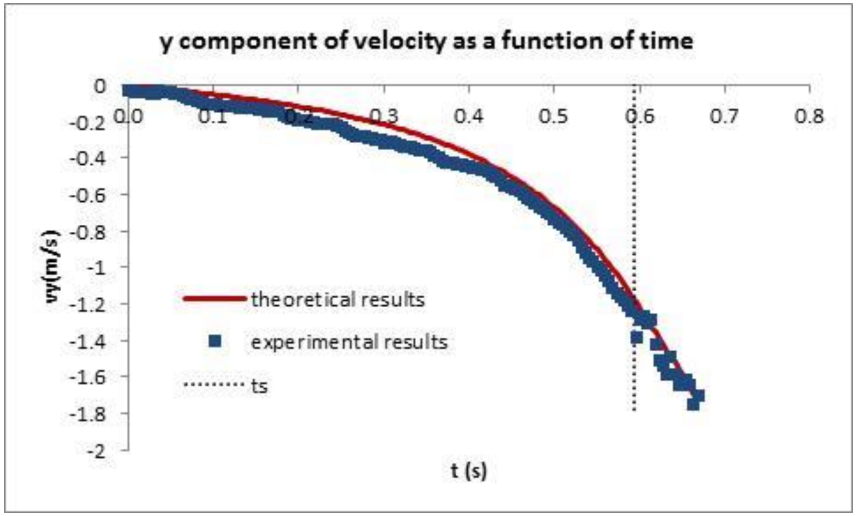


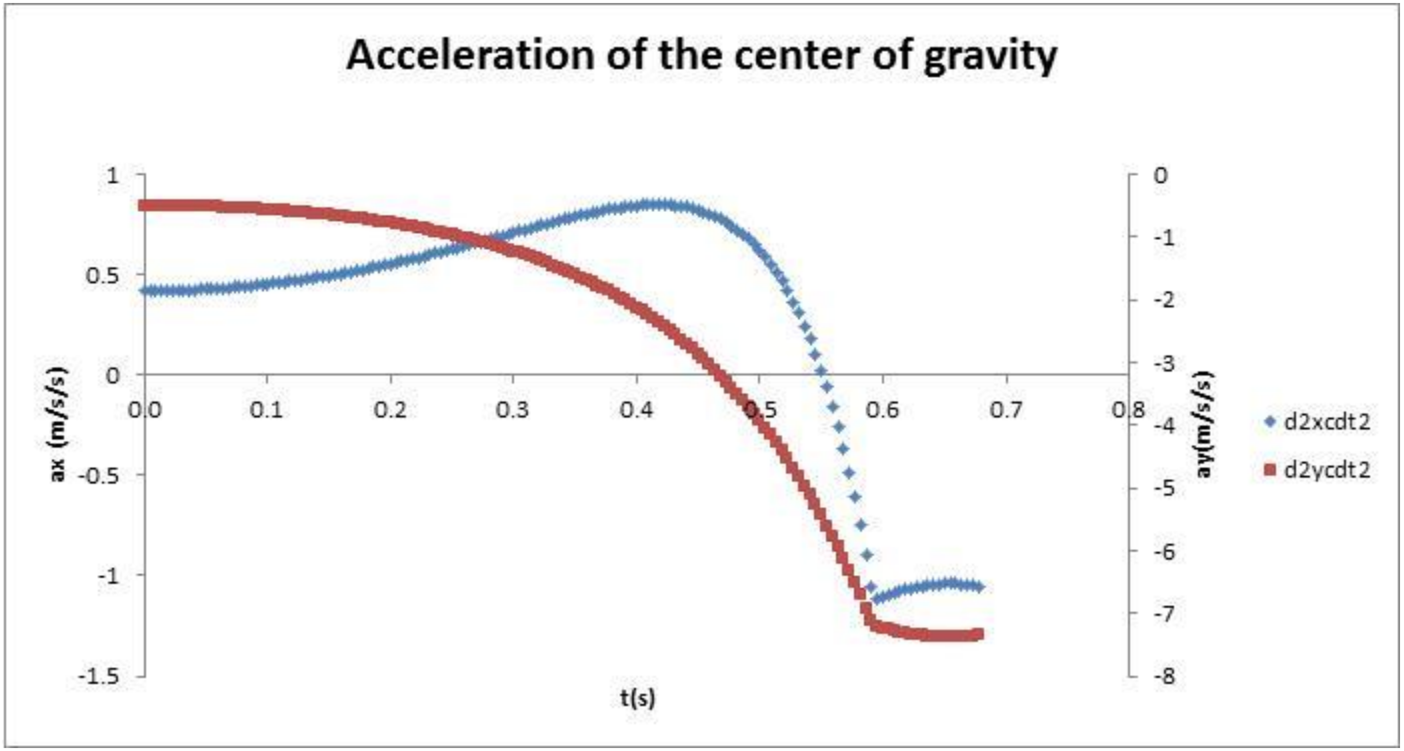


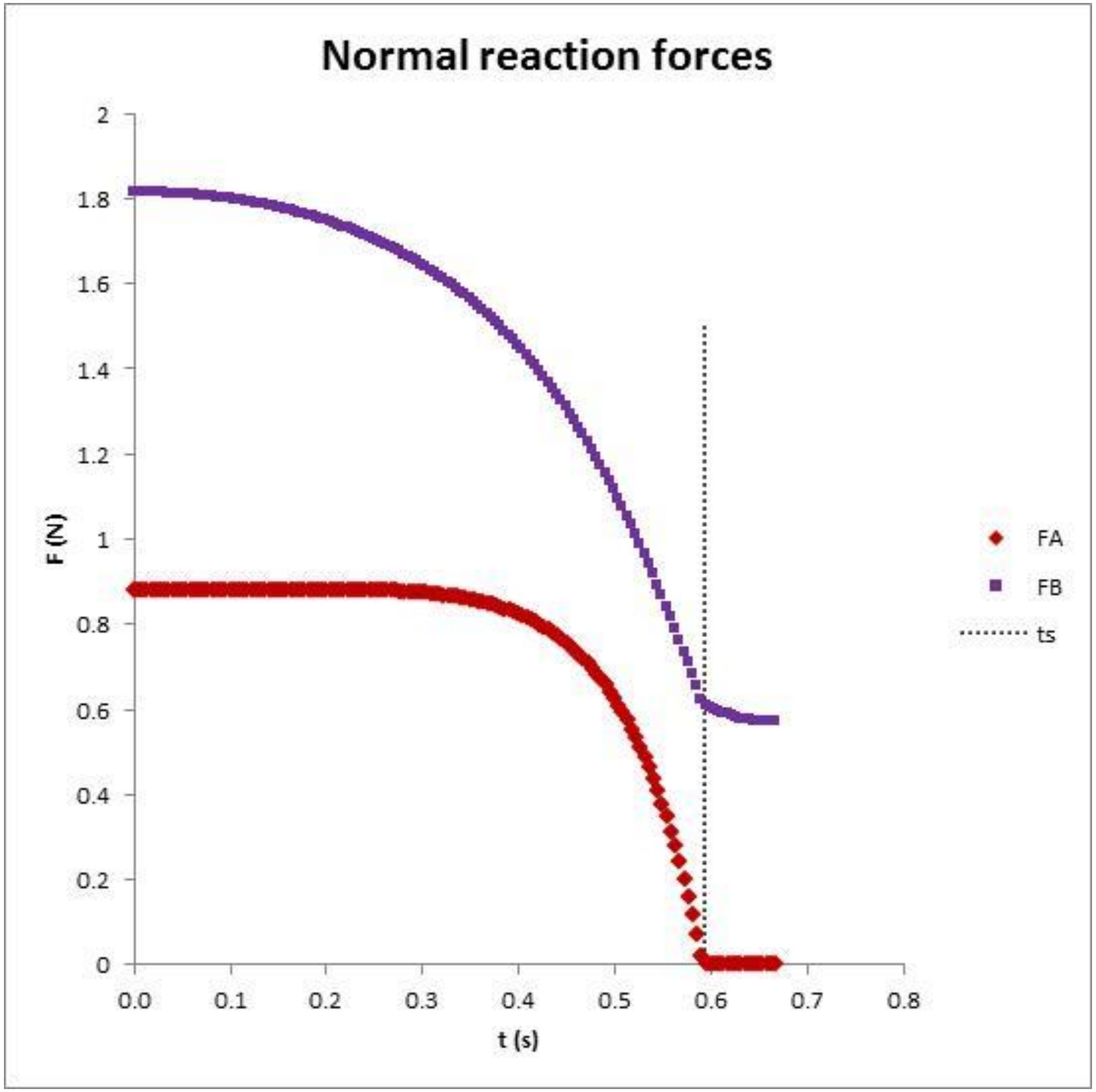


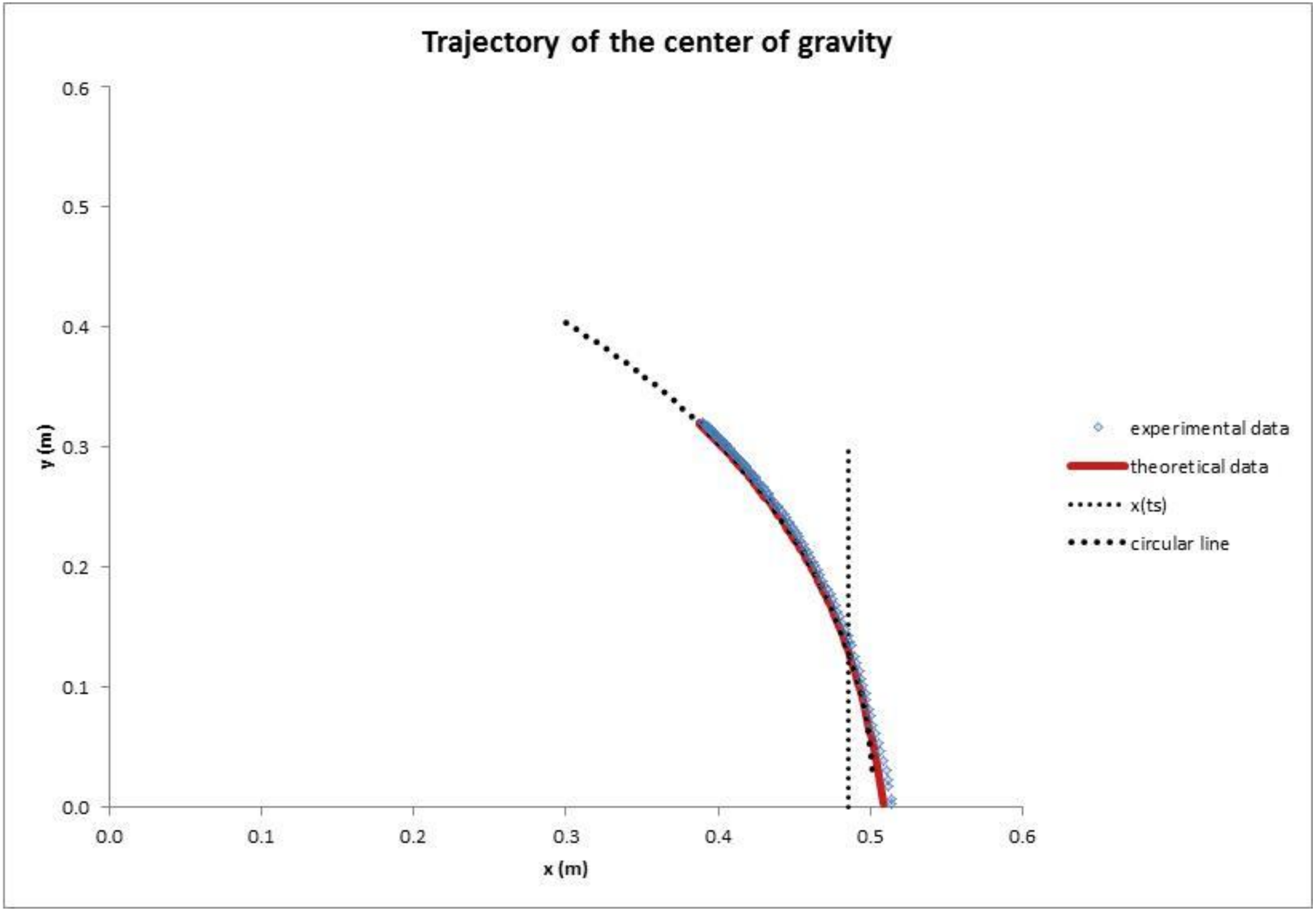
Theoretical data *versus* experimental data






Acceleration of the center of gravity
ax (m/s/s)
ay(m/s/s)
t(s)
1
0.5
0
-0.5
-1
-1.5
0
-1
-2
-3
-4
-5
-6
-7
-8
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
d2xcdt2
d2ycdt2


Normal reaction forces
2
1.8
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
F (N)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
t (s)
FA
FB
ts


Trajectory of the center of gravity
y (m)
x (m)
0.0
0.1
0.2
0.3
0.4
0.5
0.6
experimental data
theoretical data
x(ts)
circular line

Conclusions

- The experimental data are in fairly good agreement with theoretical results and, in particular, the observation of a **maximum of the x component of the center of gravity velocity**, just before the ladder loses contact with the wall.
- The comparison between the video of the experimental work, with **the simulation** of the experiment using software GeoGebra, has a **great potential in physics education** and gives a new approach for teaching mechanics introductory physics courses.

GeoGebra simulation

- References:

- [1] Mendelson, KS. 1995. Statics of a ladder leaning against a rough wall. American Journal of Physics. 63(2): 148-150.
- [2] Mario Belloni , A Simple Demonstration for the Static Ladder Problem , Phys. Teach. **46** , 503 (2008)
- [3] David Morin, Introductory Classical Mechanics, Cambridge U P (2004)
- [4] Yehuda Salu, Revisiting the Ladder on a Wall Problem , Phys. Teach. 49, 289 (2011);